Markov Localization and Bayes’ Rule

You are building a robot to monitor the Crescent at SCHS. The robot uses infra-red markers in the ceiling that it can detect with some certainty. You would like to calculate the probability $P($marker$|reading)$ to be under a certain marker given a sensor reading and information about how the robot has moved. You can find the probability $P($marker$|reading)$, or $P(M|R)$, using Bayes Rule:

$$P(M|R) = \frac{P(R|M)P(M)}{P(R)},$$

where $P(M|R)$ is what we are trying to find, $P(R|M)$ is the likelihood that robot’s reading of the marker is correct, and $P(R)$ is a normalizing term which ensures all the probabilities add to 1. In this case $P(R)$ is just the sum of the $P(M|R)$s for each of the four markers.

1. If the likelihood that the robot reads a marker correctly is 90%, i.e. $P(R|M) = 0.9$, the probability that the robot reads a marker incorrectly is 10%, and the probability that the robot does not see a marker when passing underneath it is 50%. Consider the picture below: a section of the Crescent with four markers. You know with certainty that the robot started to the left of marker 1 and moved from left to right in a straight line. The first reading the robot gets is ”Marker 3”. Calculate the probability that the robot is indeed underneath Marker 3. 

*Hint, you will have to find the probability the robot is under each of the four markers separately to calculate $P(R)$.*

2. Could the robot possibly be underneath Marker 4? If so, with what probability?

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Problem adapted from a problem used in *Introduction to Robotics*, taught by Dr. Nikolaus Correll at the University of Colorado