Project ECSITE Activity: Visibility Diagrams

Subject Areas: Algebra 2

Associated Unit: Robotics and Algebra 2

Time Required: 60 minutes

Group Size: <30 students

Expendable Cost per Group: US$ 0

Learning Objectives: Review how to compute the intersection of a line, use point-slope form, find the y-intersect, understand how these techniques can be used in robotic navigation

Computational Thinking Connection: Robotics

Materials List:

Per Class:

- 2 colors of yarn

Introduction/Motivation: Robots need to be able to move around a room or any open space to accomplish tasks. An example task would be driving/walking to a refrigerator to get a drink for a human. In order to do this, the robot must plan a path from A to B. One basic method of achieving this is with visibility graphs.

Vocabulary Definitions:

visibility graph: a graph of intervisible locations, typically for a set of points and obstacles in the Euclidean plane. Each node in the graph represents a point location, and each edge represents a visible connection between them.

Procedure: this is the outline I used for the lesson:

Outline

- Why: robot wants to navigate the 2D plane
- How: y=mx+b and slope formula are useful
- Walle wants to walk around
- Question: how can a computer do this?
- 1. Detect obstacles:
  - Pre-built map
- SLAM type thing
- Computer vision algorithm: **edge detection**
- 2. Draw straight lines that don’t intersect an obstacle
  - White board: living room becoming **decomposed**
- How to account for robots with **mass**, and **irregularly shaped obstacles**
  - Image
- How to **represent** a graph
  - what an **adjacency list** is
- **GROUP ACTIVITY**: turn the whole classroom into a visibility graph
  - Groups of 3-5 people stand around tables in the classroom, creating a sudo-outline of the table with their bodies
  - Each table represents an obstacle
  - Have a start and a goal “person”
  - Throw yarn between each person in straight lines, but not over any table
  - Once graph is built, take a different colored yarn and find the optimal path and overlay it on the classroom visibility graph
- **MATH TIME**
  - How does a computer detect if two points are visible to each other?
  - For each pair of points, check if every other **line segment intersects**
- Steps to solve:
  - See notebook
- Shortcut?
  - \( x = \frac{(b_2 - b_1)}{(m_1 - m_2)} \)
- Computation difficulty
  - More advanced algorithms for solving this problem
- **Example application**

**MATH PROCEDURES**
- Draw a quadrilateral around the 2 obstacles
- List out pairs of coordinates for each of the lines in the polygon
- For each line, attempt to connect the two points to the end points of all other line segments.
- Do not connect if there is an obstacle in the way
- Check if the two lines have a point of intersection within their domain
  \( y = mx + b \)
\[ m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \]
\[ b = y - mx \]
\[ y = \left( \frac{(y_2 - y_1)}{(x_2 - x_1)} \right)x + \left( y_1 - \left( \frac{(y_2 - y_1)}{(x_2 - x_1)} \right)x_1 \right) \]

\[ y = m_a x + b_a \] if \( x_{1a} < x < x_{2a} \)
\[ y = m_b x + b_b \] if \( x_{1b} < x < x_{2b} \)

\[ m_1 x + b_1 = m_2 x + b_2 \]
\[ (m_1 - m_2)x + b_1 = b_2 \]
\[ x = \frac{(b_2 - b_1)}{(m_1 - m_2)} \]

Is \( x \) within bounds of both limits?
- \( x_{1a} < x < x_{2a} \)
- \( x_{1b} < x < x_{2b} \)

Is so, there is an obstacle in the way and we can't draw the line!

**Safety Issues:** There is thrown yarn, but yarn isn't really dangerous

**Attachments:**
- GK12_Presentation4.pptx
- Formulas.pdf
- Latex Source for Formulas.pdf
- activity_demo.jpg

**Adaptations and Extensions:** n/a

**Assessment:**

1st lesson of the quarter went great, I had an interactive activity where students thrown yarn around to demonstrate how a visibility graph is constructed, I talked about my personal and research interests as a grad student, showed some cool robot videos and finally had students do some robotic “obstacle avoidance” calculations by finding the intersection point of two lines. Students were very well behaved and most were attentive, despite the fact it was a sub day and Alex was not present.

**References Used:**

http://en.wikipedia.org/wiki/Visibility_graph
Contributors:
Dave Coleman

Supporting Program(s):
NSF and ECSITE